XVI Seminario Técnico de Fianzas y Crédito

Premiums and Reserves –Actuarial Fundamentals

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Content

- **Technical Fundamentals of Premiums**
- **Technical Fundamentals of Reserves**

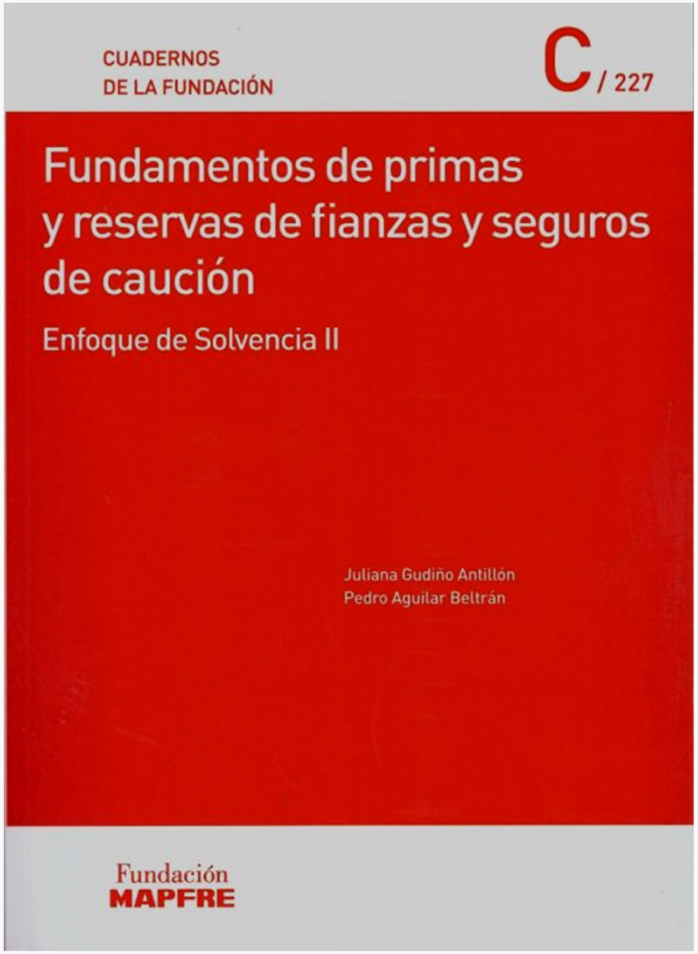
Background:

- Some features of surety bonds risk are quite different to classical insurance features.
- Actuarial classical literature is not appropriate for estimating premiums and reserves.
- Special actuarial techniques are needed to estimate premiums and reserves.
- As a result of a long researching actuarial work based on the surety bonds practices, w developed some actuarial formulas for premium and reserve estimation.
- Also, it was identified the most recommended way to compilate data for the estimati mentioned above, and the basic way of risk monitoring needed to know and manage risk.

Reference source:

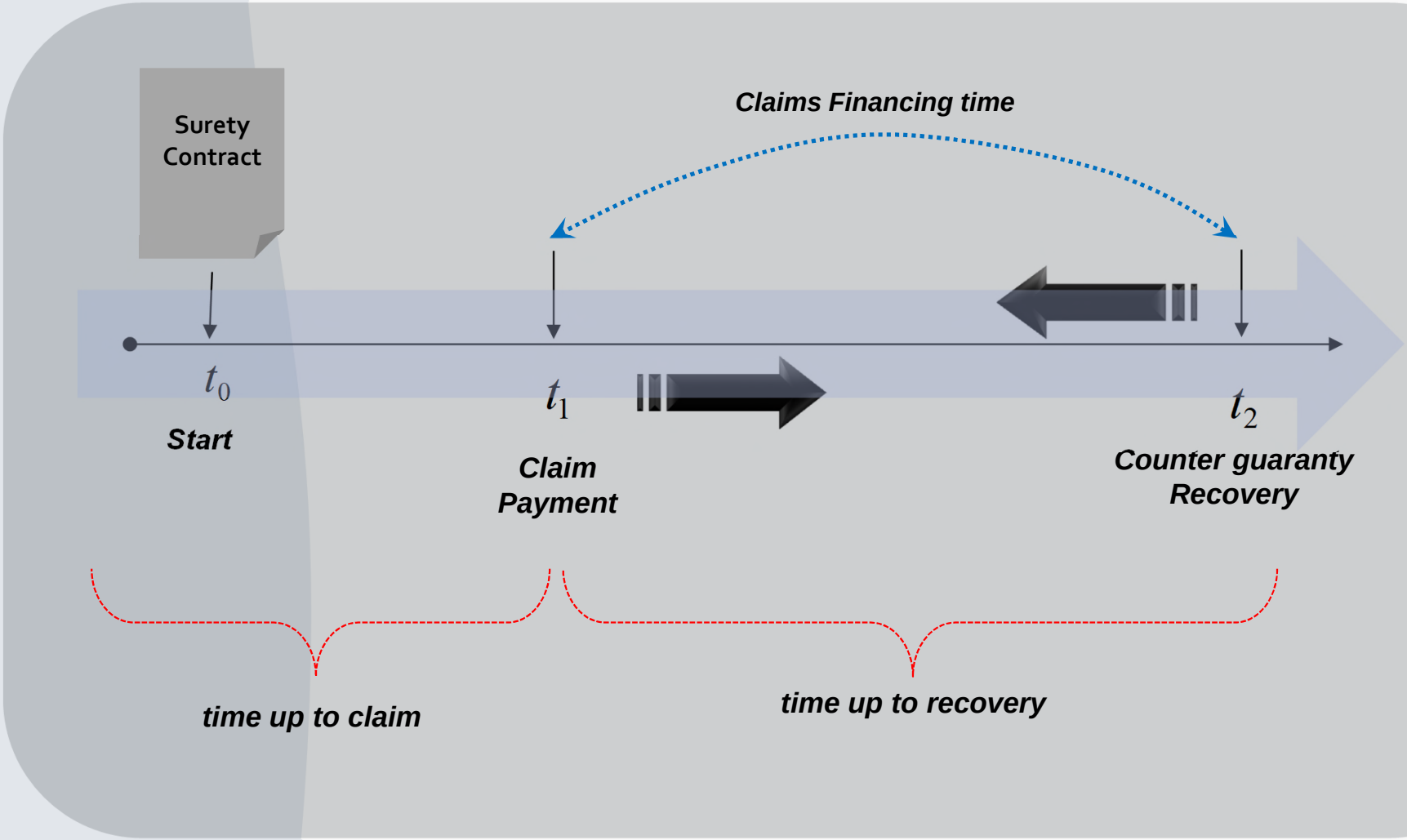
- Some of the actuarial elements shown below are based in the content of the Mapfre book “Fundamentos de Primas y Reservas de Fianzas y Seguros de Caución (2019)”.

- Mapfre Foundation has provided the means for diffusion of the research about the actuarial fundamentals of surety bonds premium and reserves.



Technical Fundamentals of Premiums

Dynamic process along of the life period of a surety bond:



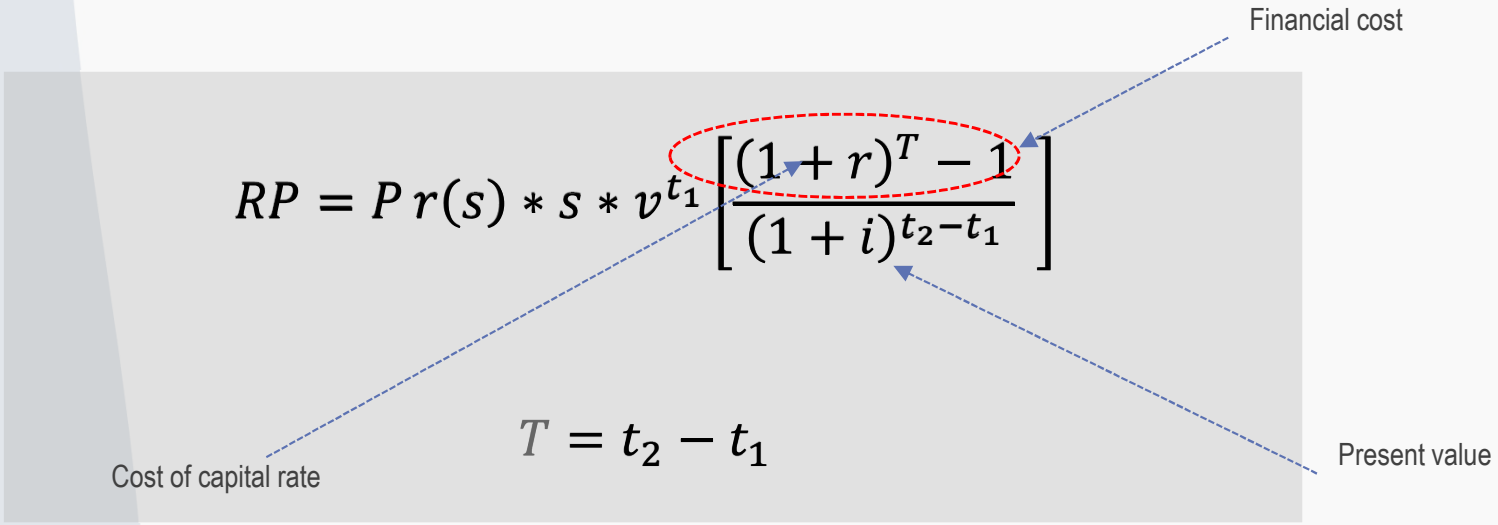
Technical Fundamentals of Premiums

Technical elements of sureties bond risk premium:

- $Pr(s)$: Probability of claims
- s: expected value of claims
- ε: ratio of counterparty recovery
- r: cost of capital rate (capital for financing payment, up to recovery)
- t_2: expected time to counterparty recovery in case of claim
- t_1: time up to claim
- T: time of claim financing period
- i: interest rate

Technical Fundamentals of Premiums

Technical risk premium formula under assumption of total recovery of claims:



- $P\,r(s)$: Probability of claims
- s: expected value of claims
- ε: ratio of counterparty recovery
- r: cost of capital rate (capital for financing payment, up to recovery)
- t_2: expected time to counterparty recovery in case of claim
- t_1: time up to claim
- T: time from the claim up to counterparty recovery
- i: interest rate

Technical Fundamentals of Premiums

Technical risk premium formula under assumption of partial recovery of claims:

$$PR = P\,r(s) * s * v^{t_1}\left[\frac{(1+r)^T - 1}{(1+i)^{t_2 - t_1}}(1-\varepsilon) + \varepsilon\right]$$

$$T = t_2 - t_1$$

Ratio of unrec
claims

- $P\,r(s)$: Probability of claims
- s: expected value of claims
- ε: ratio of counterparty recovery
- r: cost of capital rate (capital for financing payment up to recovery)
- t_2: expected time to counterparty recovery in case of claim
- t_1: time up to claim
- T: time from the claim up to counterparty recovery
- i: interest rate

Claims/Assured Amount ratio (ω):

Type of Surety	ω
Fidelity	0.0162
Judicial	0.0080
Contract and commercial	0.0022
Credit	0.0088

$$PR = \omega * v^{t_1}\left[\frac{(1+r)^T - 1}{(1+i)^{t_2 - t_1}}(1-\varepsilon) + \varepsilon\right]$$

Recoveries/Claims ratio (ε):

Type of Surety	ε
Fidelity	NA
Judicial	67.34%
Contract and commercial	71.14%
Credit	50.20%

$$PR = \omega * v^{t_1}\left[\frac{(1+r)^T - 1}{(1+i)^{t_2 - t_1}}(1-\varepsilon) + \varepsilon\right]$$

Technical Fundamentals of Premiums

Recoveries/Claims ratio (ε):

Type of Surety	ε
Fidelity	NA
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Year of payment	Claims paid	Claims recoveries															
		0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
2001	421,432,870	41,448,830	15,938,836	6,744,719	3,280,197	1,562,546	1,052,090	1,314,921	1,820,756	3,243,179	203,975	386,706	129,025	7,600	0	131,287	
2002	720,824,961	214,557,019	19,766,999	6,705,721	6,839,464	2,303,376	4,332,200	24,771,646	1,309,934	1,737,771	705,774	374,694	1,194,468	75,982	340,414		
2003	763,472,844	111,259,658	24,912,662	12,302,218	5,109,597	4,963,689	8,614,347	3,593,826	1,170,469	2,362,137	4,100,303	2,530,649	6,472,383	1,955,959			
2004	550,661,280	102,200,313	35,535,905	17,313,585	8,645,943	37,574,959	2,650,674	2,193,883	2,215,096	971,299	707,218	504,984	147,818				
2005	788,027,404	65,294,539	15,275,140	10,995,001	24,609,427	4,260,859	5,811,495	16,086,694	7,729,849	836,312	4,670,046	1,444,036					
2006	594,397,180	79,952,605	17,782,595	6,379,749	4,190,060	8,293,329	3,677,210	12,208,256	13,680,511	13,165,822	183,897						
2007	440,172,918	111,981,386	14,778,680	8,560,394	3,838,987	1,120,566	1,212,712	1,186,739	2,952,454	3,102,968	0						
2008	757,189,698	45,978,876	32,958,722	7,022,808	6,059,482	6,168,370	4,794,432	18,581,351	1,204,692	0	0						
2009	736,245,800	76,111,025	30,115,480	13,223,274	3,753,178	42,622,545	30,935,256	970,459	12,212,708	600,000	3,494						
2010	824,835,677	116,273,779	27,495,780	13,811,030	24,075,856	5,634,139	4,711,114	1,703,866	2,486,833	490,511							
2011	627,471,138	73,742,983	8,586,081	6,587,334	3,405,271	1,326,718	649,537	499,801	3,032,498								
2012	678,620,513	74,202,460	33,255,256	3,686,985	19,217,363	6,166,496	4,477,529	541,003									
2013	2,125,419,395	1,345,256,946	13,464,030	5,428,007	4,272,092	5,851,814	6,627,409										
2014	782,162,100	140,792,409	20,258,503	9,468,716	3,251,288	4,256,044											
2015	746,869,681	64,224,035	36,257,535	22,620,708	2,960,299												
2016	1,475,223,706	183,317,915	28,332,571	26,789,108													
2017	1,169,549,154	233,987,911	51,205,101														
2018	1,356,079,520	210,356,958															

Net Premium Formula (PT):

$$PT = Risk\ Premium\ (RP) + Expenses$$

$$PT = \left[\frac{RP}{1 - AE - AC - UM}\right] + C$$

RP: risk premium
AE: percentage of administrative expenses
AC: percentage of acquisition expenses
UM: percentage of utility margin
C: fixed operating cost

- Technical Fundamentals of Premiums
- Technical Fundamentals of Reserves

Reserve of Policies in Force

Is a provision for covering future claims coming from policies in force.

Reserve of Policies in Force

In accordance with the Solvency II regulatory scheme, reserves must be estimated as the sum
the best estimate liability (BEL) and the risk margin (MR):

$$Reserve = BEL + MR$$

$$BEL = BEL_c + BEL_{ae}$$

BEL_c: $Expected\ Future\ Claims$

BEL_{ae}: $Expected\ Future\ Administrative\ Expenses$

For traditional insurance products the estimation of this reserve is basic and well known, bu
the case of surety bonds the common method is not applicable.

Reserve of Policies in Force

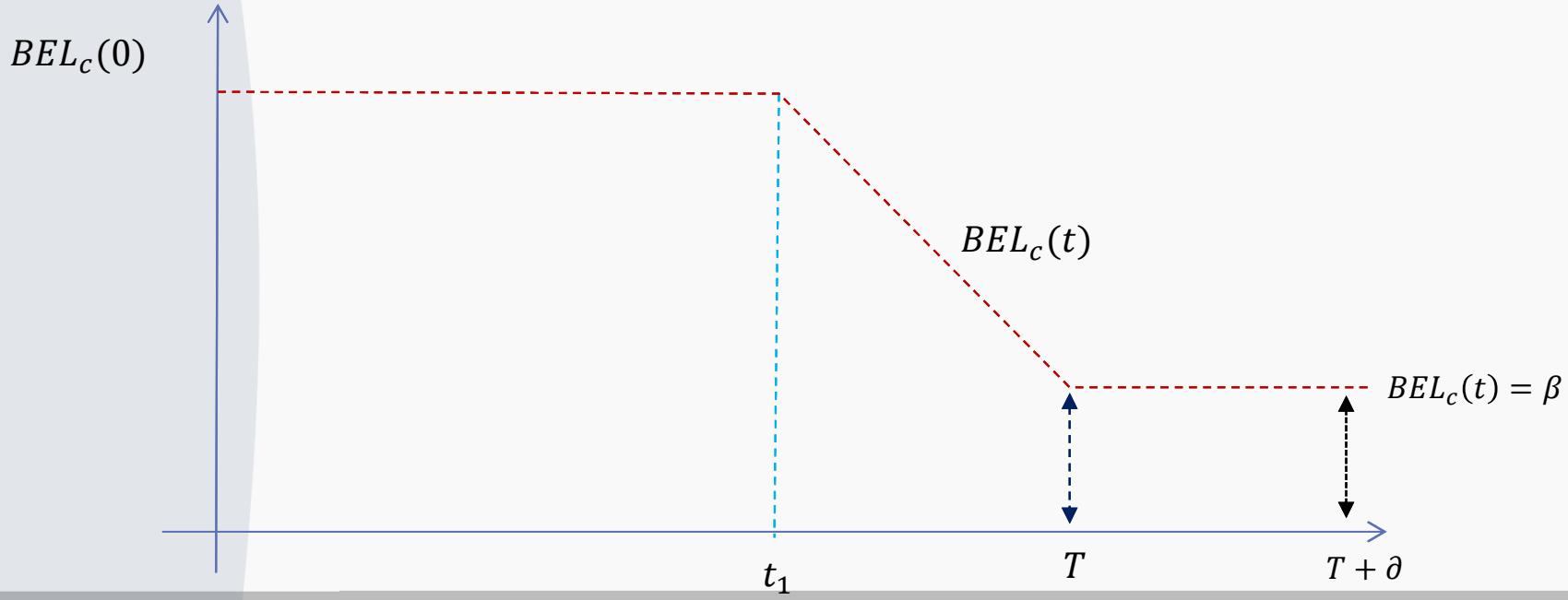
Then, what are the main differences between traditional insurance reserves and sur
bonds reserves?

1. There are multiannual term contracts.
2. Extensions are common in surety bonds.
3. The trend of claims is not uniform.
4. Counter guaranties provide future income fluxes.

An actuarial and practical technique to calculate the BEL is:

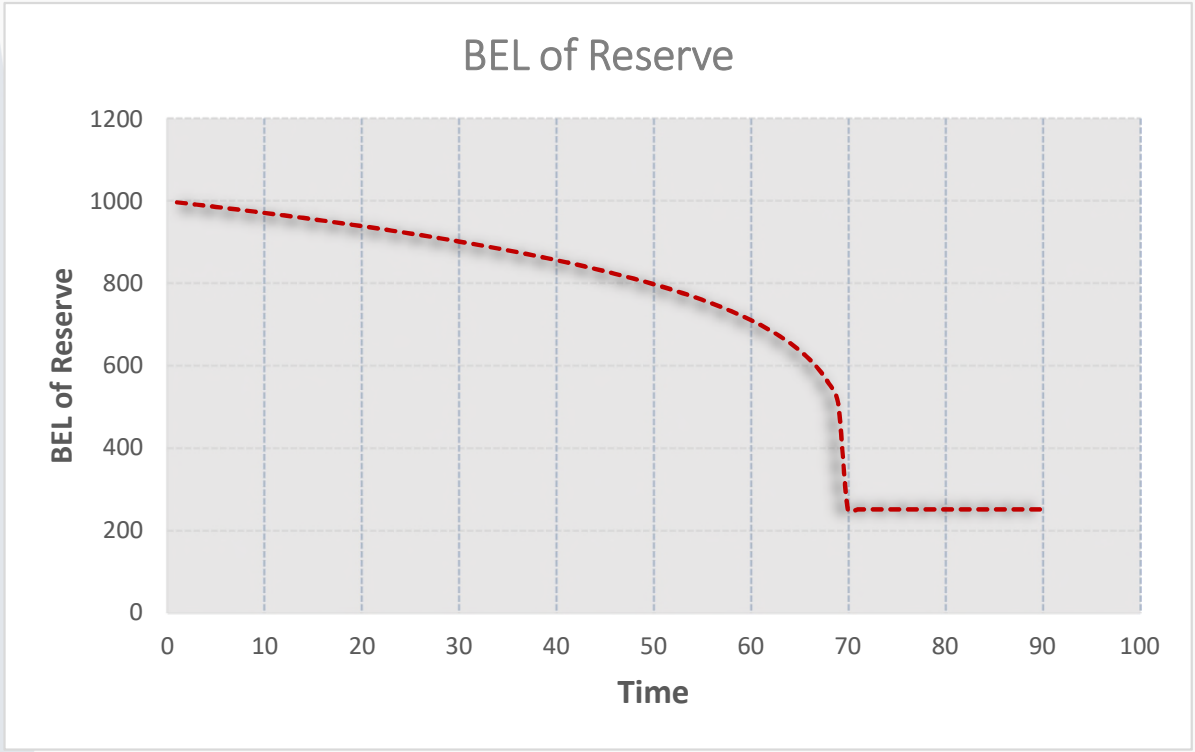
$$FD_t = \begin{cases} 1 & if \quad t < t_1 \\ \left(1 - \dfrac{\boldsymbol{t} - t_1}{\boldsymbol{T} - t_1}\right) * (1 - \beta) + \beta & if \quad t_1 < t \end{cases}$$

$$BEL_c = RP * FD_t + AE * \left(1 - \frac{t}{T}\right)$$



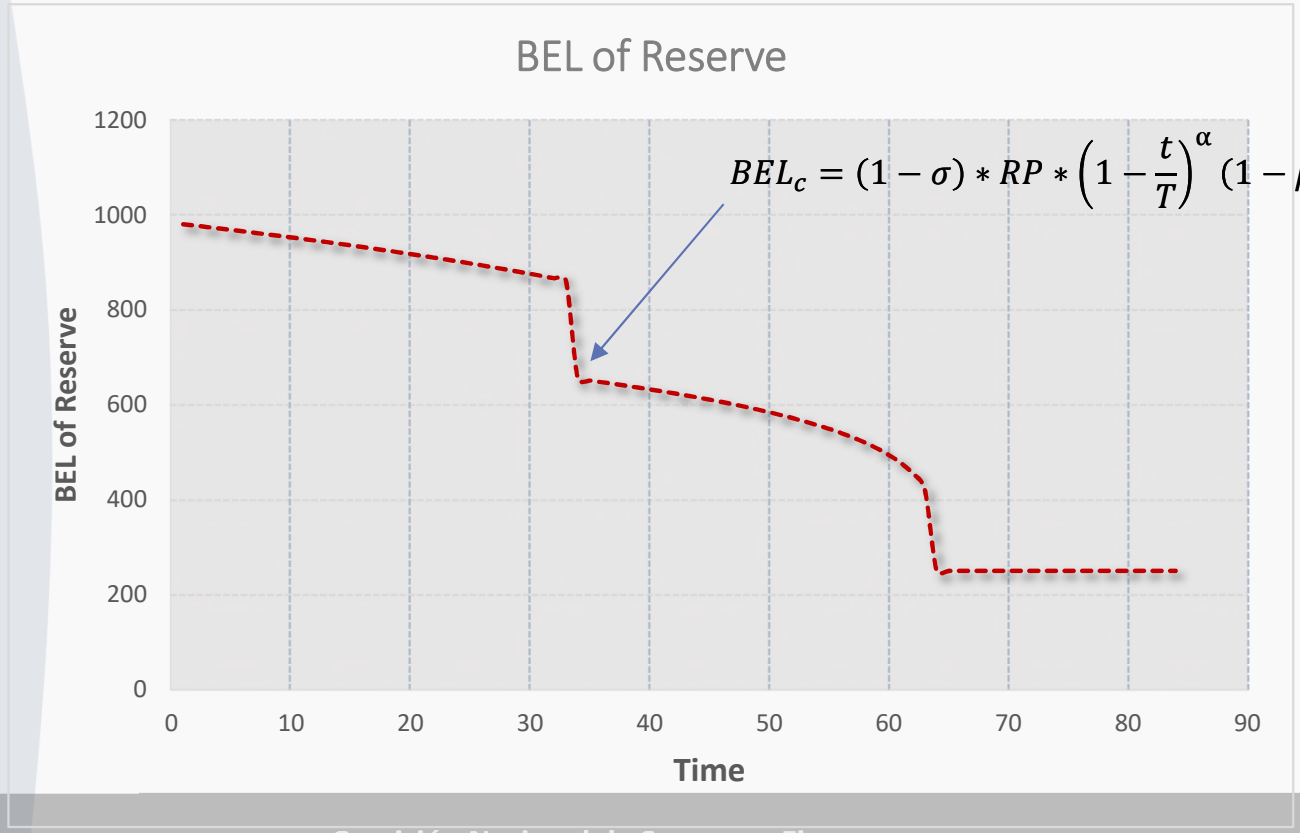
And a more theoretical technique is:

$$BEL_c = RP * \left(1 - \frac{t}{T}\right)^{\alpha} * (1 - \beta) + RP * \beta$$



In some cases of surety bonds used to guarantee construction liabilities, risk diminishes as construction process advances, so reserves must be also diminished.

$$BEL_c = (1-\sigma) * RP * \left(1 - \frac{t}{T}\right)^{\alpha} * (1-\beta) + RP * \beta$$



Administrative expenses (BEL_{ae})

As part of the reserve of policies in force, it must be estimated the future administrat expenses reserve (BEL_{ae}). The recommended technical formula to estimate the BEL_{ae} is:

$$BEL_{ae} = AE * PT * \left(1 - \frac{t}{T}\right)$$

Based on:

$$PT = \left[\frac{RP}{1 - AE - AC - UM}\right] + C$$

Risk Margin

The actuarial formula to estimate the Risk Margin (RM), at time t_0, is:

$$RM_{t_0} = r * SCR(t_0) * duration(t_0)$$

Where,

r: cost of capital rate

$SCR(t_0)$: Solvency Capital Requirement at moment t_0

$duration$: expected life time of liabilities at moment t_0

Risk Margin

Assuming that one policy is in the moment t_0 and that the cost of capital rate is 0.1, then the is:

$$RM_{t_0} = 0.1 * RCS * duration$$

$$= 0.1 * RCS * \int_{t_0}^{T} \left(1 - \frac{t}{T}\right)^{\alpha} dt$$

Special Reserve

Is a special provision for covering future claims coming from a catastrophic event

Special Reserve (Contingency Reserve)

Phenomenology of surety bond claims is characterized by long periods of low number of cla
followed by a suddenly big cumulus of claims produced, sometimes, by economical crisis. So
face this kind of scenarios is recommended to set up a special reserve.

The special reserve must be set up by mean of cumulative annual contributions with a part of
profits and reserve-earned interests.

$$Special\ Reserve(t) = Special\ Reserve(t-1) + interest(t) + contribution(t)$$

$$Contribution(t) = \varphi * Profits\ (t)$$

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